Application Note: Wireless SAW stress/strain sensor

1 Introduction

In this article, firstly, we will explain the principle of SAW sensors. We will then describe how to realize wireless measurements with SAW Sensors. The second part of this document is focused on the application of stress measurements. A fixed-free flexural beam is used to demonstrate the performances of wireless SAW sensors as wireless stress gauge. The advantages of such device lies within the comparison between the theoretical and experimental values.

2 Principle of SAW stress sensors

Surface Acoustic Waves are elastic waves launched by the fields generated at the interdigital transducers (IDTs) acting through the piezoelectric effect (Figure 1). Figure 2 shows Rayleigh mode wave.

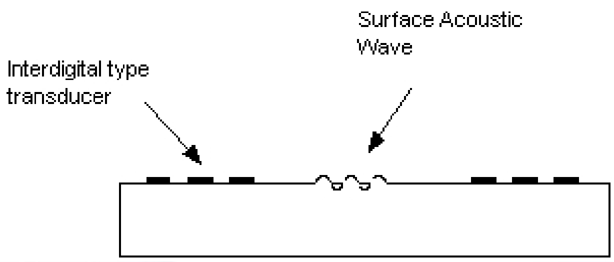

Fig. 1 – Representation of the generation and propagation of surface acoustic wave.

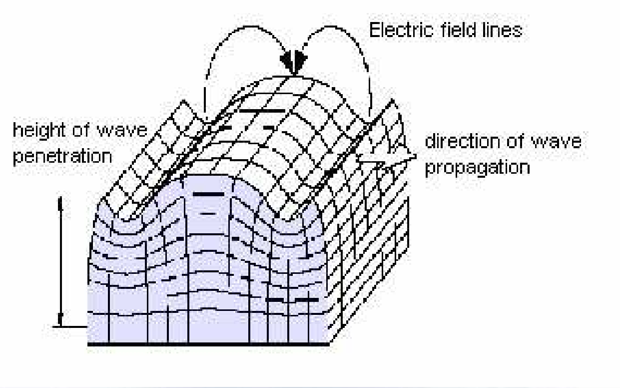

Fig. 2 – Rayleigh mode wave.

Using this property of generating acoustic waves with IDTs on a piezoelectric substrate (i.e. quartz, langasite, lithium niobate, tantalate, …), we can design two kinds of devices : resonators and delay lines (Figures 3 and 4).

- a resonator is composed of IDTs in the center of the structure and reflecting gratings or electrodes on both sides of the IDTs. The IDT is a bi-directional structure, which means the energy propagates on both sides at the same intensity. The reflecting gratings or electrodes reflect the energy produced by the IDT. So we obtain a resonant cavity characterised by its resonant frequency.
- a delay line is composed of IDTs at one side of the device and reflecting gratings or electrodes at the other side. The IDTs generate an impulse wave which propagates to the electrodes. The impulse wave is reflected by the electrodes or reflecting gratings to the IDTs. So we obtain a device that measures the propagating time of an impulse.

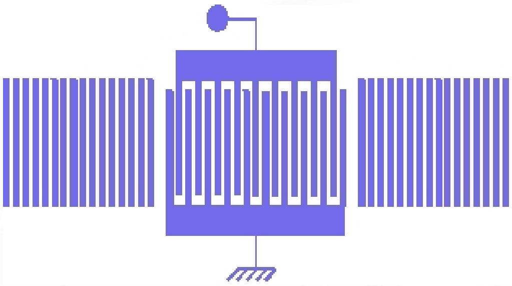

Fig. 3 – SAW Resonator.

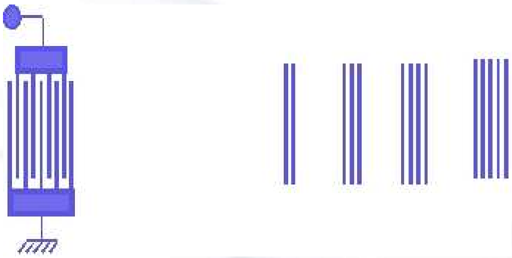

Fig. 4 – SAW Delay Line.

The wave propagation speed of the SAW device depends on the piezoelectric materials, the design of the IDT and the environment (Temperature, Pressure, Stress, ...). To design a SAW sensor, some design parameters of the SAW resonator can be optimized to shift with a specific parameter, in our case, the stress.

The SAW stress sensor used in this application note is based on a resonator structure (Figure 5). It is made of a quartz substrate with aluminium electrodes as the IDTs realized by photolithography. The thickness of the electrodes is about 0.1 µm. The period of the IDTs is approximately 3.5 µm in order to have a resonant frequency at 433 MHz.

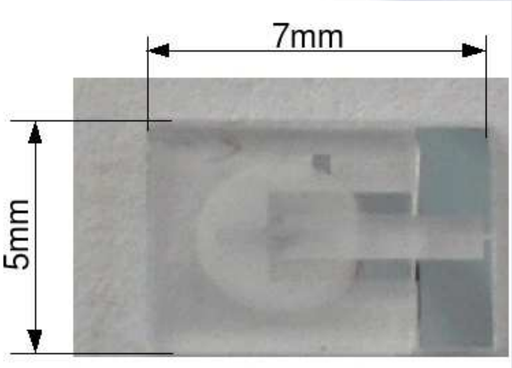

Fig. 5 – SAW Stress sensor used in this application.

3 Principle of wireless measurement using SAW sensors

A wireless measurement is based on an interrogation unit (also composed of an emission part and an reception part i.e. E/R) and a SAW sensor. Each other has an adapted antenna to the working frequency (ISM Band 433 MHz, 868 MHz, 2.45 GHz, ...).

The principle of wireless interrogation is the following (Figure 6) :

- The emitter part of the interrogation unit sends an interrogation signal (temporal impulse) to the SAW sensor.
- If the resonant frequency of the signal sent is closed enougth to the resonant frequency of the SAW sensor then the SAW sensor will resonate after a charging time. It occurs a stable oscillating regime at the resonant frequency of the SAW device. This resonant frequency is proportional to the speed of surface acoustic wave which depends on the physical phenomenons (temperature, stress, pressure, ...) seen by the SAW sensor.
- The sensor sends back a signal at its own resonant frequency which contains the information of the physical phenomenons measured.
- The receiver part of the interrogation unit detects a piece of or the complete SAW signal and extracts the information of the physical phenomenons measured using an adapted signal processing.

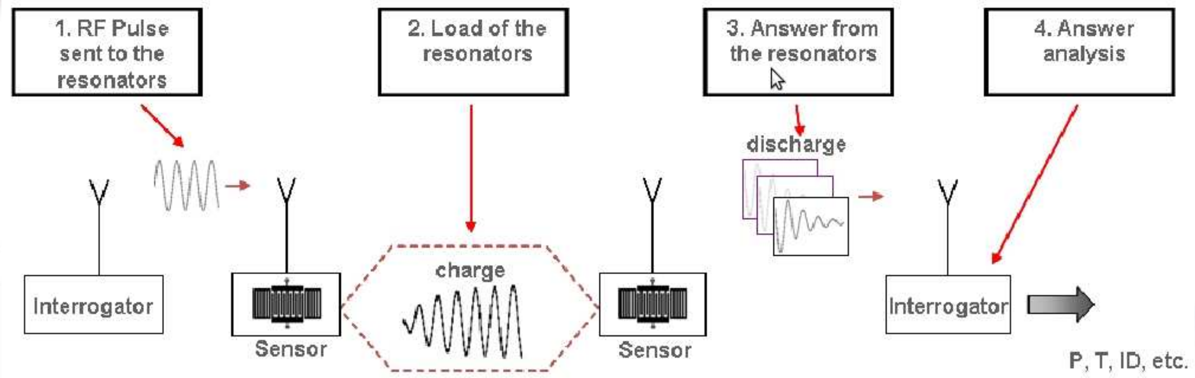

Fig. 6 – Principle of wireless interrogation.

Fig. 7 – Interrogation unit used in the application note.

4 Description of the application

In this application note, we focus on the stress of flexural beam. Figure 8 represents a fixed-free flexural beam. The position where the stress is determined theoretically and measured experimentally with the SAW sensor is also shown.

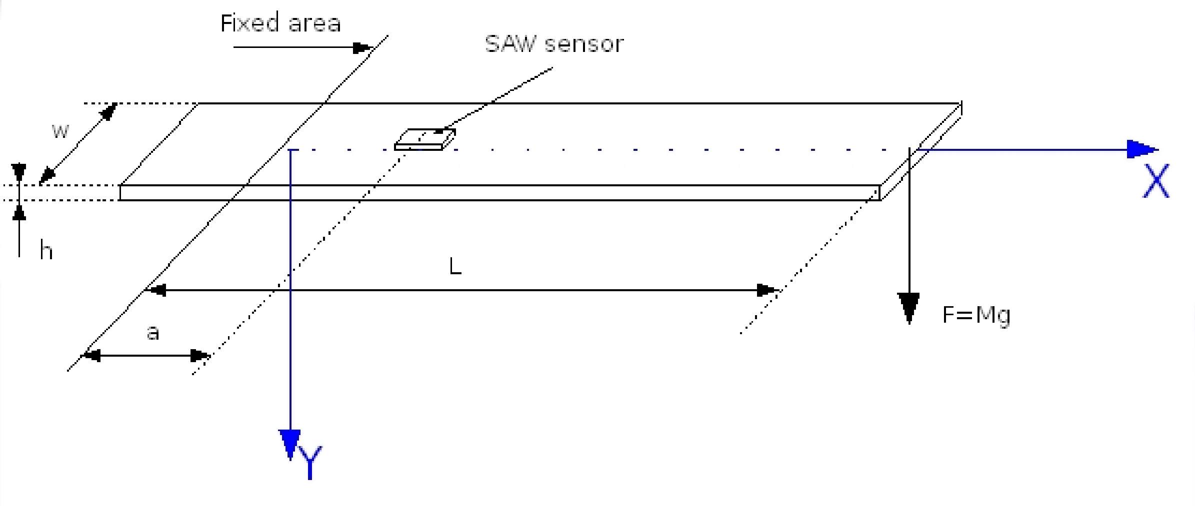

FIG. 8 – Fixed-Free flexural beam.

Using the hypothesis of Bernouilli :

- the length of the beam is huge compared to the thickness and the width ($L \gg h$ and $L \gg w$).
- the cross-section of the beam stays orthogonal to the neutral fiber.
- the weight of the beam and the sensor is neglected.

The expression of the first principle of the mechanic is :

$$\begin{aligned} \frac{\partial T}{\partial x} + p(x) &= 0 \\ \frac{\partial M}{\partial x} + q(x) + T &= 0 \end{aligned}$$

with T the shear force , M the bending moment, $p(x) = 0$, $q(x) = 0$.

The boundary conditions of the beam is fixed at one side and a force F is applied at the other side. Hence the bending moment is :

$$M = F\,(x - L)$$

Furthermore, the curvature of the beam (κ) is equal to the second derivative of the deflection and can be linked to the bending moment (M) and the flexural rigidity (EI where E is the young modulus and I is the moment of inertia).

$$\kappa = -\frac{EI}{M} = -EI\frac{\partial^2 v}{\partial x^2}$$

Moreover, the deformation (dL) is a function of the curvature of the beam and the strain (ϵ_{xx}) a function of the deformation.

$$\begin{aligned} \frac{L}{\kappa} &= -\frac{dL}{y} \\ \epsilon_{xx} &= \frac{dL}{L} \end{aligned}$$

The Hooke's law links the strain to the stress.

$$\sigma_{xx} = E\epsilon_{xx}$$

Hence, at the position of the sensor ($x = a$), the stress on the top surface of the beam (i.e. $y = -\frac{h}{2}$) is :

$$\sigma_{xx} = \frac{Fh(L-a)}{2I} \quad \text{with} \quad F = mg \tag{1}$$

Now we will calculate the natural frequencies of the beam then we apply the first principle of mechanic to the beam :

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} + \rho S\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \tag{2}$$

Using the separation of variables $v(x,t) = V(x)f(t)$, the equation (2) can be written :

$$\frac{\partial^4 V(x)}{\partial x^4} - \alpha^4 V(x) = 0 \quad \text{with} \quad \alpha^4 = \omega^2\frac{\rho S}{EI}$$

The general solution of this equation is a linear combination of trigonometric equations :

$$V(x) = C_1\sinh(\alpha x) + C_2\cosh(\alpha x) + C_3\sin(\alpha x) + C_4\cos(\alpha x) \tag{3}$$

The boundary conditions let the determination of the constants C_1, C_2, C_3 and C_4 through the solution of the frequency equation. Then the resonant frequency of the beam is :

$$\omega = \frac{\lambda^2}{L^2}\sqrt{\frac{EI}{\rho S}} \quad \text{with} \quad \lambda = \alpha L \ \text{ solution of the frequency equation.}$$

For a fixed-free beam used in the application note, the frequency equation is :

$$1 + \cos(\lambda) + \cosh(\lambda) = 0$$

with $\lambda = 1.8751$, 4.6841, 7.8548, 10.996, 14.135, ...as solutions.

The dimensions of the beam used in this application note are :
- $L = 170$ mm
- $h = 1.7$ mm
- $w = 19$ mm
- $\rho = 1910$ kg/m^3
- $E = 17$ GPa

Then the three first natural frequencies of the beam are :

Mode	Frequency (Hz)
1	28.34
2	176.9
3	497.46

The theoretical calculus does not take into account the damping of the structure.

5 Measurements

Now let's do the experiments : we will use a SAW sensor and the interrogation unit to realize wireless stress measurements of the beam. The SAW sensor is glued to the beam at the position a (Figure 8). The reliability of the measure is strongly dependant on the glue.

Figure 9 represents the fixed-free beam with the SAW sensor and its adapted antenna.

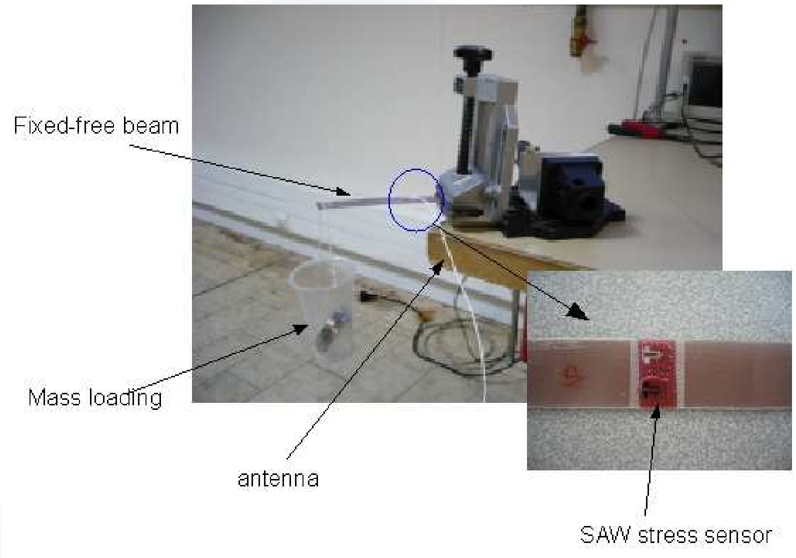

FIG. 9 – Fixed-free beam with the SAW sensor and its adapted antenna.

The experiments consist in measuring the stress due to the force applied at the end of the beam. At the moment the force is removed, the beam's vibration is measured (see below).

5.1 Calibration

To achieve good measurements of the stress, it is necessary to calibrate the sensor on the application. A SAW sensor is characterised by a resonant frequency, a stress sensitivity and a temperature dependence.

The properties of the SAW Sensor used in this application are :
- $f_0 = 433.8$ MHz
- $s = 14$ ppm/MPa (without glue) [10 ppm/MPa sticked with "cyanoacrylate" as glue]
- $TCF_1 = -1.68$ ppm/°C
- $TCF_2 = -34.147$ ppb/°C^2

The stress sensitivity (σ) is proportional to the resonant frequency (f_0) :

$$\sigma = \frac{1}{f(\sigma = 0, T_0)s} f(\sigma, T) - \frac{1}{s}$$

The parameter $\frac{1}{s}$ is equivalent to an offset. It is modified by the antenna impedance and the environment (SAW sensor glued to the beam). It is then necessary to settle the offset before the first using. The offset is configured by adjusting this parameter to obtain a zero value when no stress is applied.
Furthermore the resonant frequency is dependant on the temperature according the following expression :

$$f(\sigma, T) = f(\sigma, T_0)\left(1 + TCF_1(T - T_0) + TCF_2(T - T_0)^2\right) \quad \text{with} \;\; T_0 = 25^{\circ}\text{C}$$

All measurements realized in this application are obtained at room temperature (i.e. $T = T_0$). As a consequence, there is no shift of resonant frequency due to the temperature.

5.2 Measurements

Using the fixed-free beam described previously, we realize different measurements. The first measurements consist of measuring differents stress due to different mass loading at the free end of the beam. Figure 10 represents the results obtained. We can notice a good correlation between theoretical and experimental values. The noise on the measurement signal for 14.5 MPa and 21.7 MPa stress is the vibration of the beam. This vibration are produce at each variation of stress. It demonstrates the dynamic of the SAW Sensor.

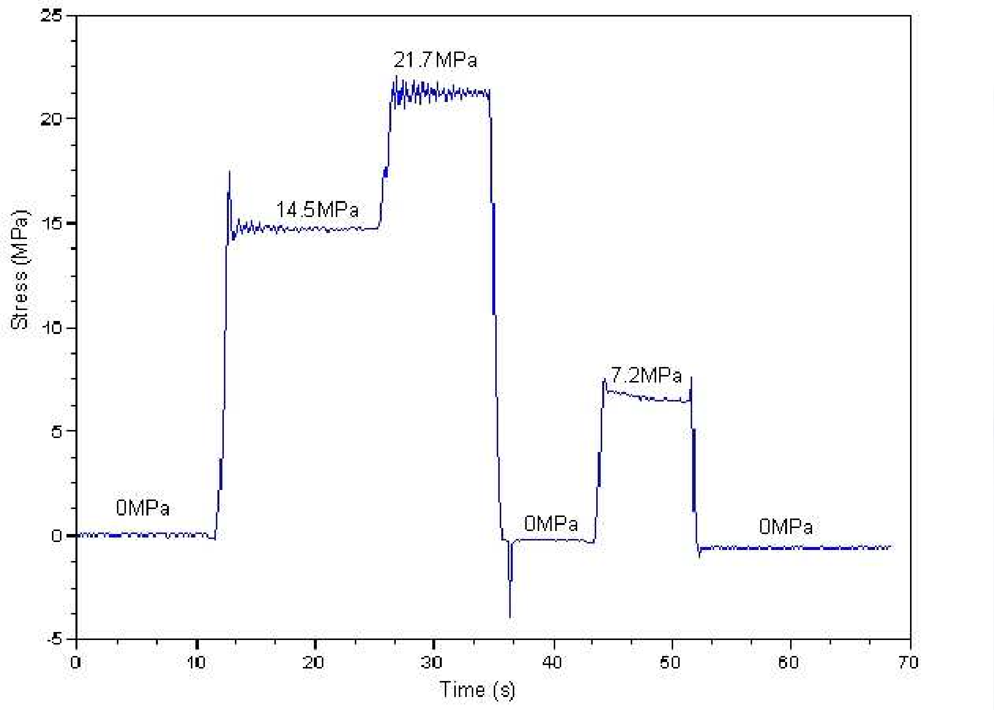

Fig. 10 – Stress measurement.

Figure 11 represents the response of the beam to an impulse at the free end.

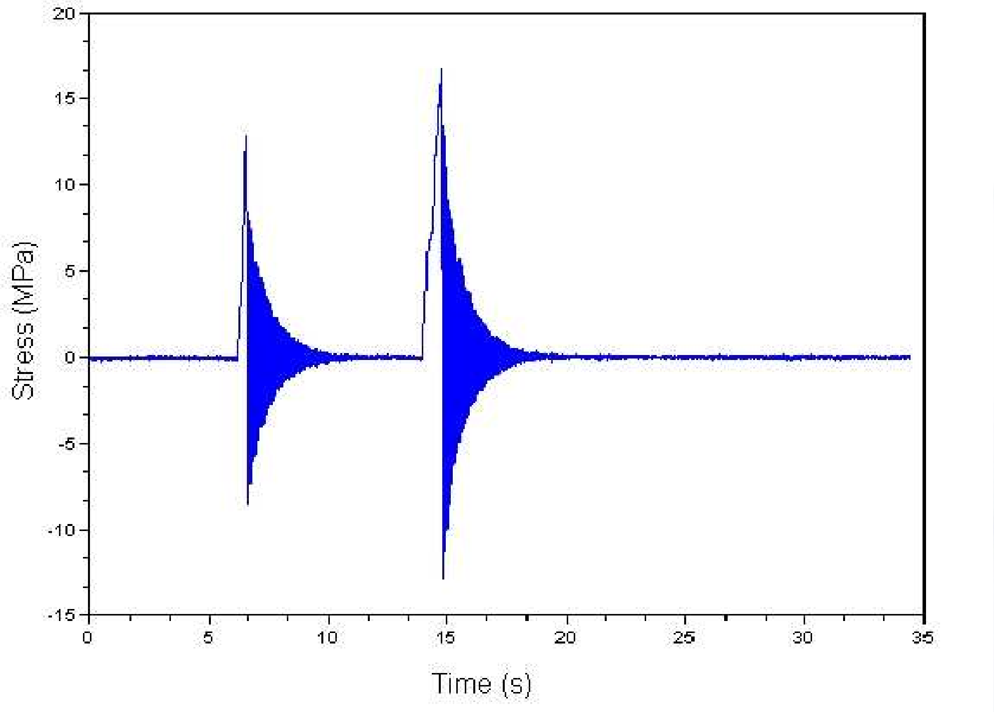

Fig. 11 – Beam response to an impulse at the free end.

Figure 12 shows a zoom of the the response of the beam. The resonant frequency of the beam can be determine from that mesaurement, directly reading the period of the vibration or using the Fourier transform of the signal. Figure 13 shows the result of Fourier transform.

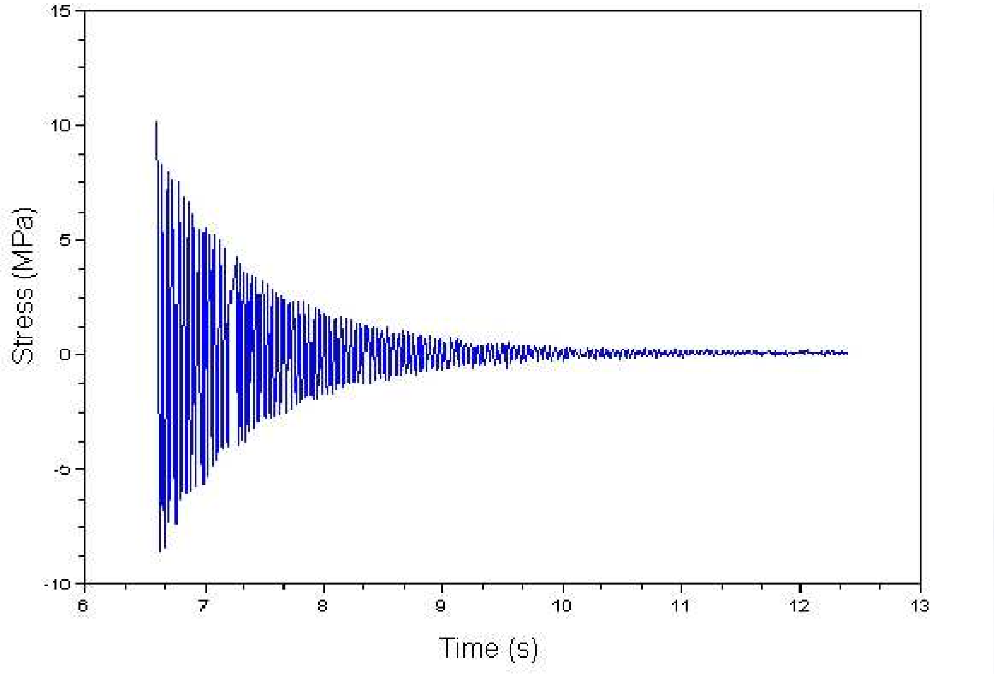

Fig. 12 – Vibration of the beam.

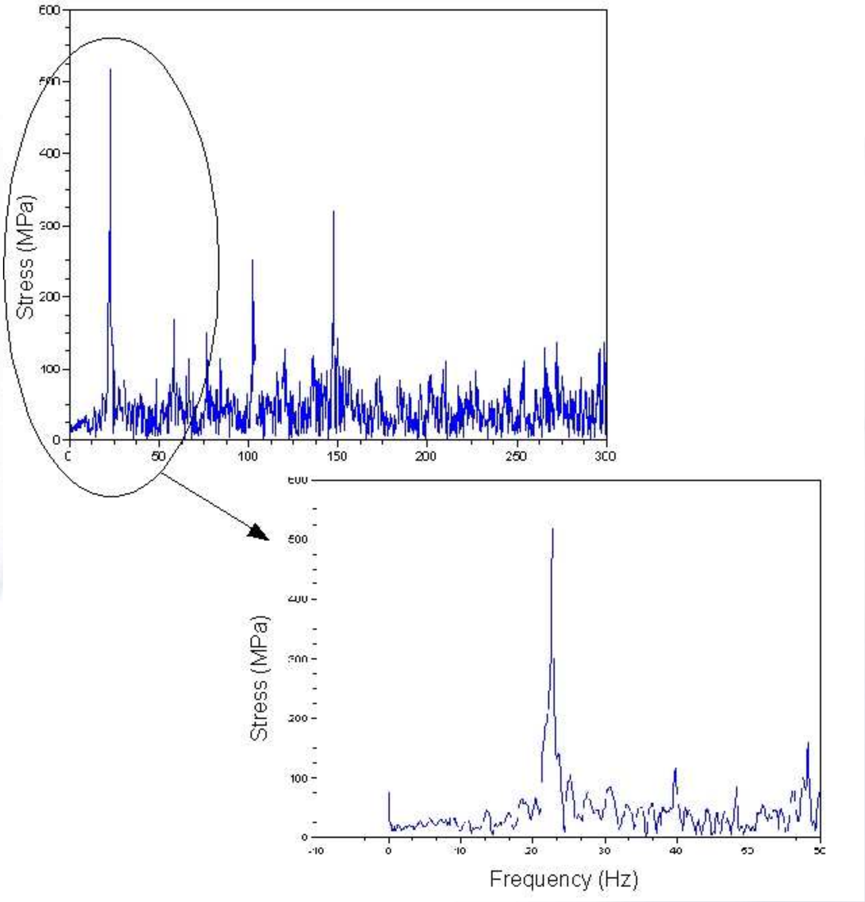

Fig. 13 – Fourier Transformation of the measurement.

We notice on the Figure 13 that the resonant frequency of the signal is at 24 Hz which is close to the theoretical value calculated previously in that application note.

6 Conclusion

To conclude in this application note is shown how to use and settle a wireless SAW stress sensor in order to measure a mechanical stress. The simple flexural beam lets to demonstrate easily the agreement between theoretical and experimental values of the static stress and dynamic stress. In this application note is also demonstrate the accuracy of the SAW stress sensor.